Aromaticity, the DaVinci Code and the Golden Section


## A Regular Pentagon has Internal Angles of $108^{\circ}$

The sum of all supplementary angles in any polygon equals $360^{\circ}$.

In a regular pentagon each supplementary angle equals $72^{\circ}$.

Thus $180^{\circ}-72^{\circ}=108^{\circ}$

Where Baeyer went wrong.

A regular pentagon can be inscribed in a circle.

Connecting alternate vertices of a pentagon produces the pentacle, a figure imbued with mysticism.


## Angles Subtending a Chord (Arc)

Two line segments that subtend the same chord and meet on the circle have the same angle.


## Similar Isosceles Triangles

The interior angles $\left(108^{\circ}\right)$ of the pentagon are trisected into angles $\theta=36^{\circ}$


## The Golden Section

$\triangle \mathrm{ACD}$ is similar to $\triangle \mathrm{ABC}$ with base angles of $2 \theta$
and line $\mathrm{AC}=\mathrm{CD}=\mathrm{BD}=\mathrm{x}$

If $\mathrm{AB}=1$, then $\mathrm{AD}=1-\mathrm{x}$
$\mathrm{x} / 1-\mathrm{x}=1 / \mathrm{x}$ or $\mathrm{x}^{2}+\mathrm{x}-1=0$
$x=0.618$ and $1 / x=1.618$ for positive values.


## Cyclopentadienyl anion



Aromaticity Meets the Da Vinci Code

## Am I Aromatic?



- planar $\pi$-system
- cyclic array
- 10 double bonds; 20 electrons; $4 n$
-No!

A Closer Look




## Am I Aromatic?



- planar $\pi$-system
- cyclic array
- 11 double bonds; 22 electrons; $4 n+2$
- Yes!


## The Bee Hive

The bee can enter any cell but it must enter at cell 1 and then to subsequent contiguous cells in ascending numerical order.

| Cell | Routes |
| :---: | :---: |
| 1 | 1 |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 5 |
| 6 | 8 |
| 7 | 13 |
| 8 | 21 |



Cell 4: 1-2-4; 1-3-4; 1,2,3,4 but not 1,3,2,4
The route to a given cell is the sum of the routes to the two previous cells.

## Fibonacci Series



A series of numbers in which each number is the sum of the two preceding numbers.
" 0 ", $1,1,2,3,5,8,13,21,34,55,89,144,233$, $377,610,987,1597,2584,4181,6765,10946 \ldots$

Leonardo Pisano
Fibonacci
(~1170-1250) mouse over

The route to a given cell is the sum of the routes to the two previous cells.

## Fibonacci Series

| Fibonacci Series | $\mathbf{a} / \mathbf{b}$ | $\mathbf{b} / \mathbf{a}$ |  |
| :---: | :---: | :---: | :--- |
| 1 | 1 | 1 |  |
| 1 | 0.5 | 2 |  |
| 2 | 0.667 | 1.5 |  |
| 3 | 0.6 | 1.667 |  |
| 5 | 0.625 | 1.6 |  |
| 8 | 0.615 | 1.625 | $\mathrm{a} / \mathrm{b}=$ smaller/larger number |
| 13 | 0.619 | 1.615 |  |
| 21 | 0.618 | 1.619 | $\mathrm{~b} / \mathrm{a}=$ larger/smaller number |
| 34 | 0.618 | 1.618 |  |
| 55 | 0.618 | 1.618 |  |
| 89 | 0.618 | 1.618 | The Golden Section (Phi) |
| 144 | 0.618 | 1.618 | is the limit of the ratio b/a. |
| 233 | 0.618 | 1.618 |  |
| 377 | 0.618 | 1.618 |  |
| 610 | 0.618 | 1.618 |  |
| 987 | 0.618 | 1.618 |  |
| 1597 | 0.618 | 1.618 |  |
| 2584 | 0.618 | 1.618 |  |
| 4181 |  |  |  |

## Fibonacci Spiral and the Golden Rectangle




Leonardo' s Mona Lisa

## bort

